

MONOTONIC CONVERGENCE OF DISTRIBUTED INTERFERENCE PRICING IN WIRELESS NETWORKS

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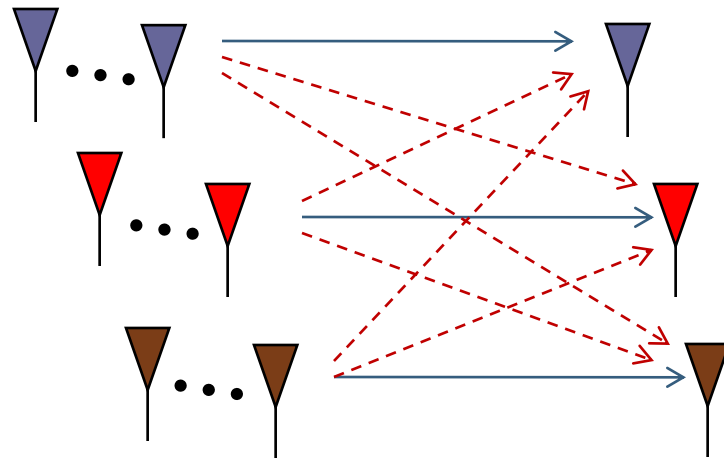
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Wireless Spectrum sharing



- Fundamental problem in wireless networking is to **share spectrum** among multiple user.
- Historically done via licensing/centralized allocation.
- Here our focus is on fully **distributed approaches**.
 - ▣ Main challenge here is **interference**.

Network Model



- Focus on network with K “users”
 - ▣ Each user = *active* transmitter/receiver pair.
 - ▣ transmitters might be equipped with multiple antennas.
 - All users transmit over a **common frequency band**
 - ▣ Initially assume users spread power over entire band.
- ⇒ **Spectrum sharing = Power allocation.**

SISO: Power Allocation Problem

- Each user i must determine **power**:

$$p_i \leq P_i$$

- Choice of all powers determines each user's **SINR**:

$$\gamma_i = \frac{h_{ii}p_i}{n_0 + \sum_{j \neq i} h_{ji}p_j}$$

- User i 's performance is measured via a **utility** $U_i(\gamma_i)$.

- Concave, increasing.

Objective

$$\max \sum U_i(\gamma_i(p_i, p_{-i}))$$

subject to: $0 \leq p_i \leq P_i \forall i.$

- Non-concave, non-separable objective.
- Want distributed solution with *limited information exchange*.
 - ▣ Baseline information: User i knows $U_i(\cdot)$, h_{ii} and γ_i
- **Simple distributed approach**: let each user selfishly optimize their own utility.
 - ▣ No information exchange.
 - ▣ Not optimal

Interference pricing

- User's pay-off needs to reflect the *externality* imposed on others due to interference.
- A simple way to incorporate this is to have users exchange *interference prices*

$$\pi_i = \frac{\partial U_i(\gamma_i)}{\partial I_i}$$

- Can calculate using only local information
 - Single price per user, broadcast to all other users.
- Given interference prices, user i updates his power by maximizing the modified pay-offs:

$$U_i(\gamma_i) - \sum_{j \neq i} \pi_j p_i h_{ij}$$

- Requires users to measure adjacent channel gains.
- *Asynchronously* update powers and interference prices.

Restricted best response updates

- Consider best response updates with the following two restrictions:
 1. At each time at most **one** user updates his power.
 2. After each power update, **all** users announce new interference prices.

- **Prop.** With these modifications, best response updates converge to a Nash equilibrium when

$$0 \leq CR_i(\gamma_i) \leq 2$$

where:

$$CR_i(\gamma_i) = -\frac{U_i''(\gamma_i)\gamma_i}{U_i'(\gamma_i)}$$

is the *coefficient of relative risk aversion* of U_i

Note: may not converge to global optimum for $CR_i \leq 1$.

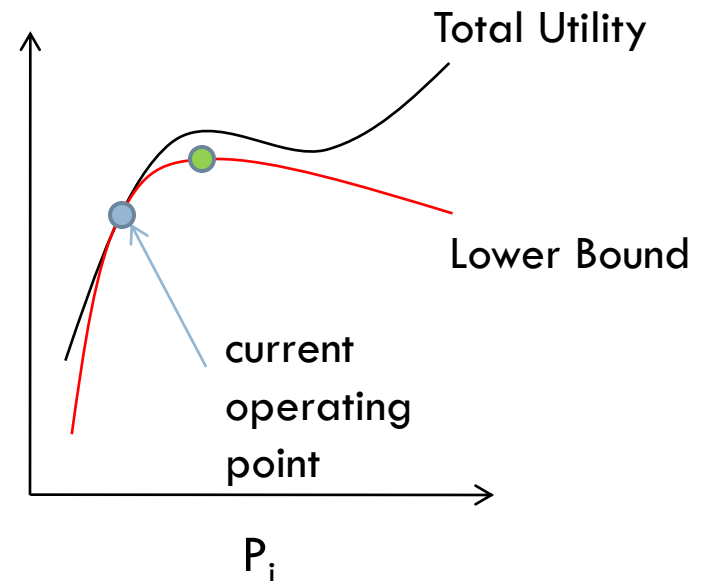
Main proof idea

- Under given assumptions, each user's utility is a **convex** function of the **total interference**.
- Interference price = the slope of tangent of utility with respect to another user's interference power at current operating point.

$\Rightarrow -\pi_i \Delta p_j h_{ji}$ lower bounds the change in user i 's utility due to a change in user j 's power.

\Rightarrow Each power update is optimizing a lower bound on the total utility, which is tight at current operating point.

\Rightarrow monotonic convergence.



MISO Model

- Suppose now that each transmitter is equipped with N transmit antennas.
- Antennas used for *beamforming*.

- \mathbf{v}_i = beamformer for user i ($\mathbf{v}_i \in \mathbb{C}^N$).

- User i 's beam must satisfy power constraint:

$$\|\mathbf{v}_i\|_2^2 \leq P_i^{\max}$$

- Choice of all beams determines each user's SINR:

$$\gamma_i = \frac{|\mathbf{v}_i^\dagger \mathbf{h}_{ii}|^2}{n_0 + \sum_{j \neq i} |\mathbf{v}_j^\dagger \mathbf{h}_{ji}|^2}$$

- Want to jointly allocate power and choose beams to maximize sum-utility.

MISO-Distributed Pricing

- Each user i announces one *interference price*

$$\pi_i = -\frac{\partial u_i}{\partial I_i} = \frac{u'_i |\mathbf{v}_i^\dagger \mathbf{h}_{ii}|^2}{(n_0 + \sum_{j \neq i} |\mathbf{v}_j^\dagger \mathbf{h}_{ji}|^2)^2}$$

- User i updates \mathbf{v}_i to maximize following pay-off:

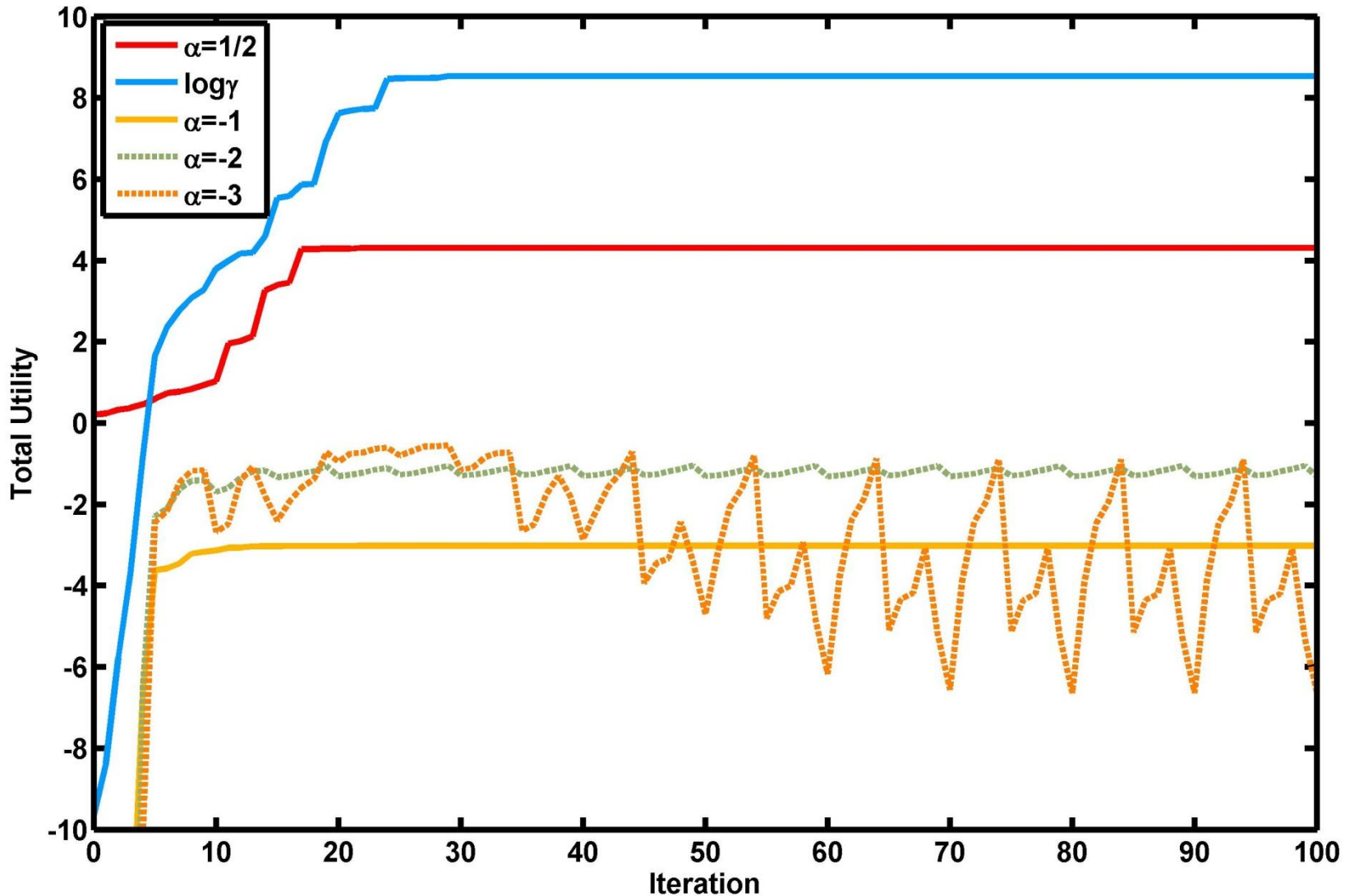
$$\begin{aligned} \max_{\mathbf{v}_i} \quad & u_i \left(\frac{|\mathbf{v}_i^\dagger \mathbf{h}_{ii}|^2}{n_0 + \sum_{j \neq i} |\mathbf{v}_j^\dagger \mathbf{h}_{ji}|^2} \right) - \sum_{j \neq i} \pi_j |\mathbf{v}_i^\dagger \mathbf{h}_{ij}|^2 \\ \text{s.t.} \quad & |\mathbf{v}_i|^2 \leq P_i^{max}. \end{aligned}$$

Again need to know adjacent gains and interference prices.

- **Proposition:** With the same modifications, best response updates converge for any number of users when

$$0 \leq CR_i(\gamma_i) \leq 2$$

Numerical Example (5 3x1 MISO users)



Conclusions

- Presented variations of distributed interference pricing algorithms for SISO and MISO links.
 - ▣ Trade-off synchronization/information exchange with class of utility function we can guarantee convergence.
 - ▣ Results can be generalized to *Multi-channel Networks*

- Open issues:
 - ▣ efficiency loss for rate utilities
 - ▣ Incentive issues.
 - ▣ Further limiting information exchange when many degrees of freedom.