Performance Analysis of a MAC Protocol in Wireless Line Networks Using Statistical Mechanics

Ka Hung Hui, Dongning Guo and Randall Berry, Dept. of EECS, Northwestern University
Martin Haenggi, Dept. of EE, University of Notre Dame

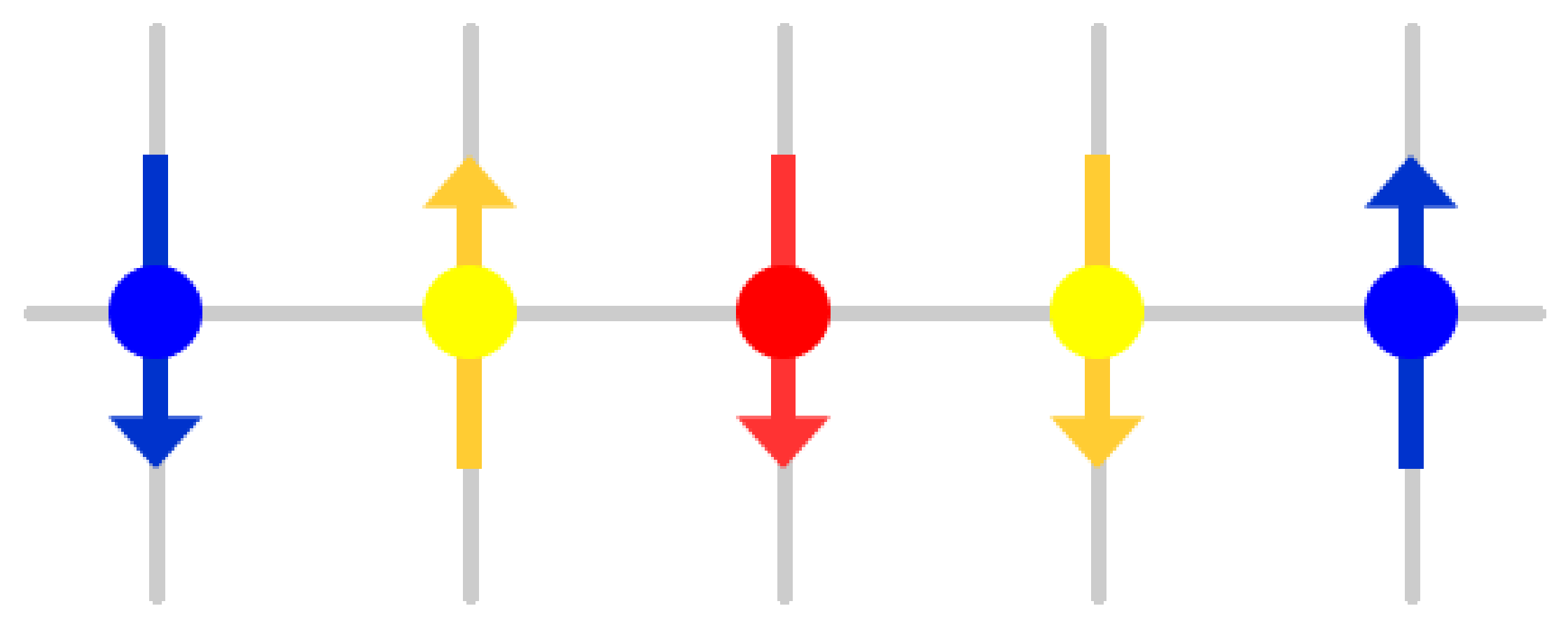
Motivation

The performance of wireless networks is limited by mutual interference between stations.

Interference is a short-range effect among 'neighboring' stations.

The resulting interaction among stations in a wireless network is similar to that found in the *Ising model* in *statistical mechanics.*

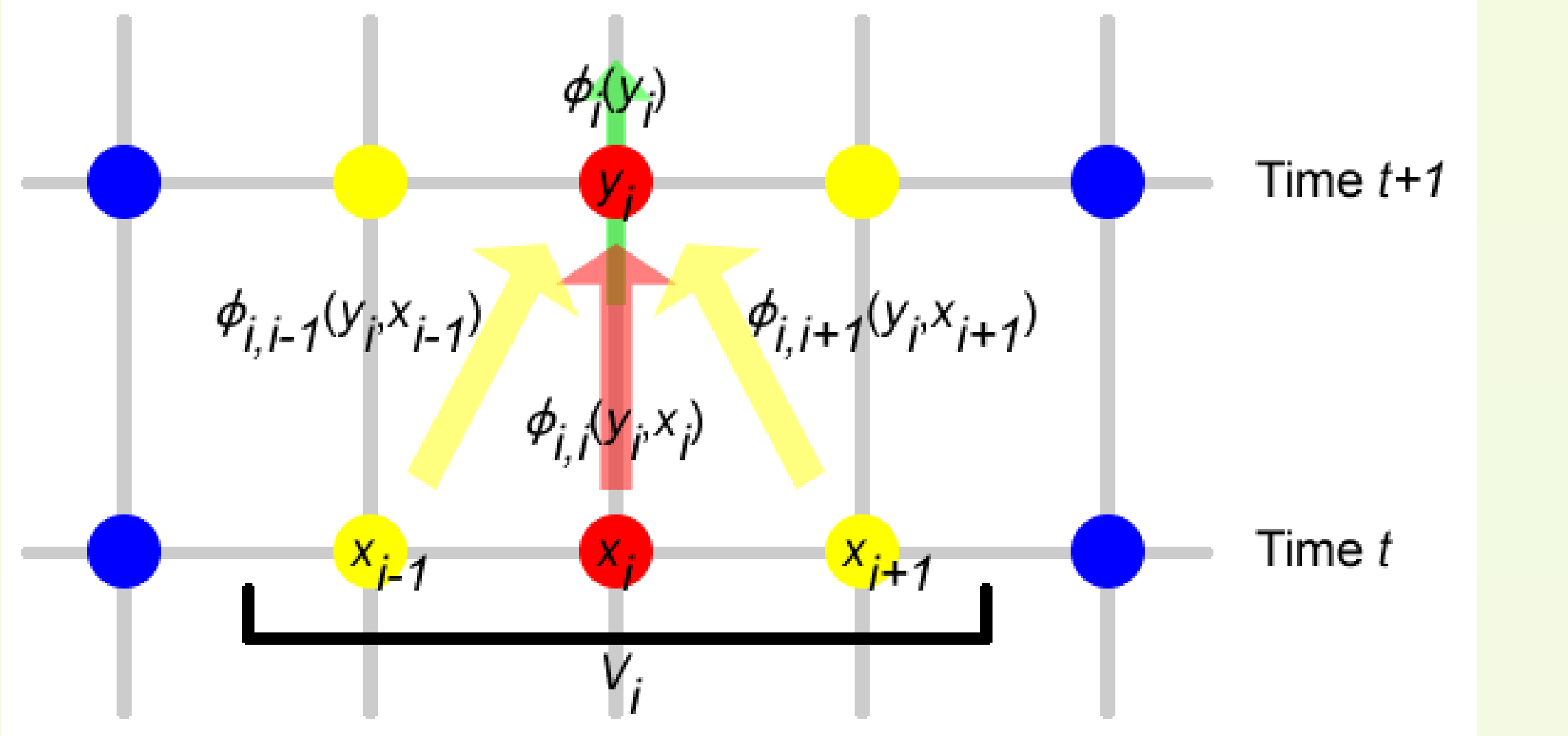
Ising Model



The Ising model was originally used to model a spin system in stationarity, where a spin system consists of a number of spins on a regular lattice.

The spins can be in 'up' or 'down' states, and their states are affected by an external magnetic field acting on them and the interactions between neighboring spins.

System Model



Spins = wireless stations
- 'Up' = transmitting
- 'Down' = idle

Each station probabilistically decides its state in the current time slot based only on the observed states of its neighbors and its own state in the previous time slot.

$$P(\mathbf{y}\mid\mathbf{x})=\prod_{i\in S}p_i(y_i\mid\mathbf{x})$$

$$p_i(y_i\mid\mathbf{x})=\frac{\exp\left(\phi_i(y_i)+\sum_{j\in V_i}\phi_{i,j}(y_i,x_j)\right)}{\sum_{z_i\in E}\exp\left(\phi_i(z_i)+\sum_{j\in V_i}\phi_{i,j}(z_i,x_j)\right)}$$

Stationary Behavior

If $\phi_{i,j}(y_i,x_j) = \phi_{j,i}(x_j,y_i)$, the stationary behavior can be represented by an Ising model.

$$\pi(\mathbf{y})=\frac{1}{Z}\exp\sum_{i\in S}\Phi_i(\mathbf{y})$$

$$\Phi_i(\mathbf{y})=\phi_i(y_i)+\ln\sum_{z_i\in E}\exp\left(\phi_i(z_i)+\sum_{j\in V_i}\phi_{i,j}(z_i,y_j)\right)$$

Performance Evaluation

$$\phi_i(y_i)=hy_i$$

$$\phi_{i,j}(y_i,x_j)=\begin{cases}Jy_ix_j & j=i-1,i+1\\ J'y_ix_j & j=i\\ 0 & \text{else}\end{cases}$$

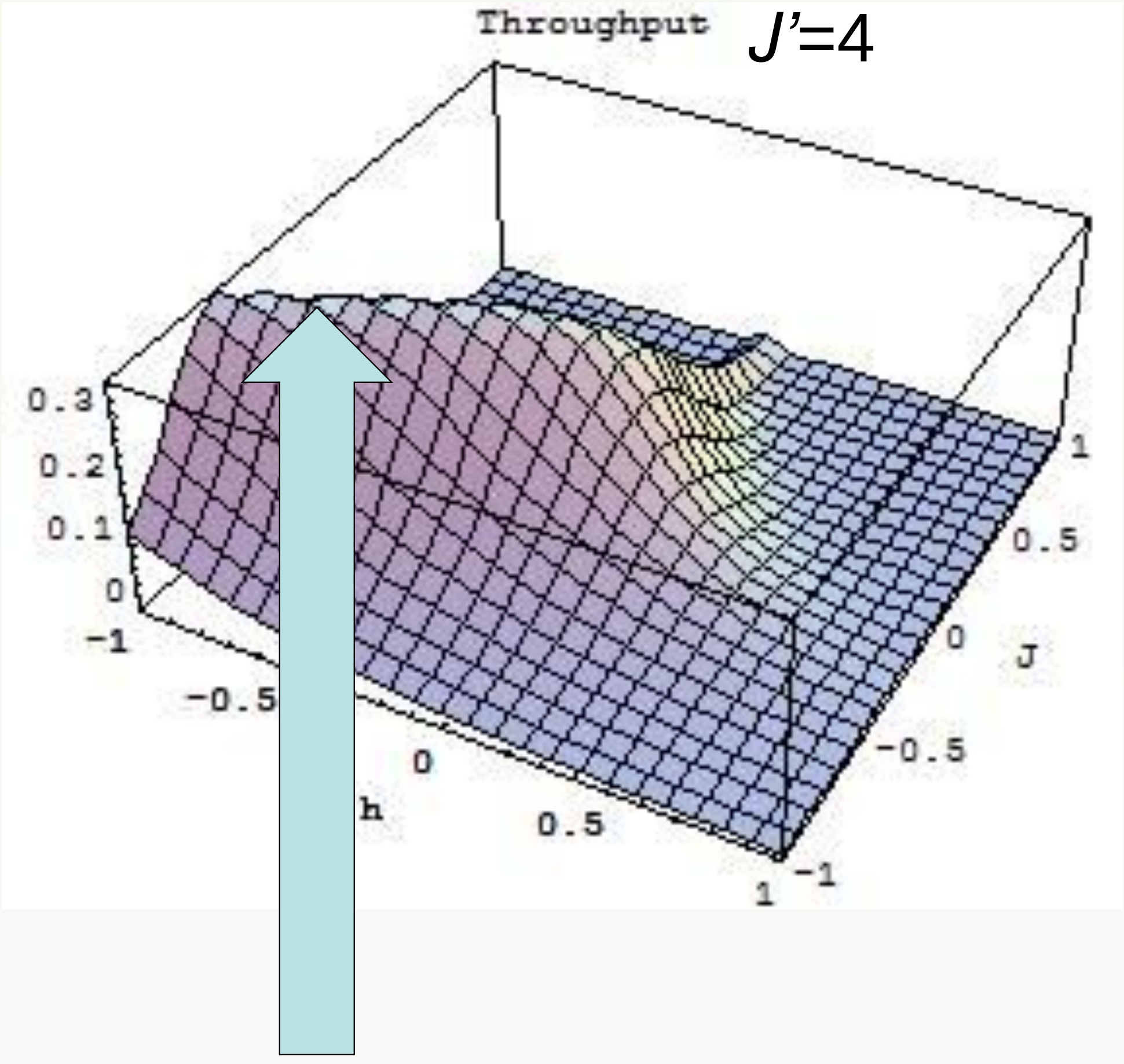
<u>Optimal (One-hop) Throughput</u>

Collision Channel: 0.3431
(Slotted ALOHA: 0.2963)
Multipacket Reception Channel: 1
(Slotted ALOHA: 0.5)

<u>Optimal Parameter Choices</u>

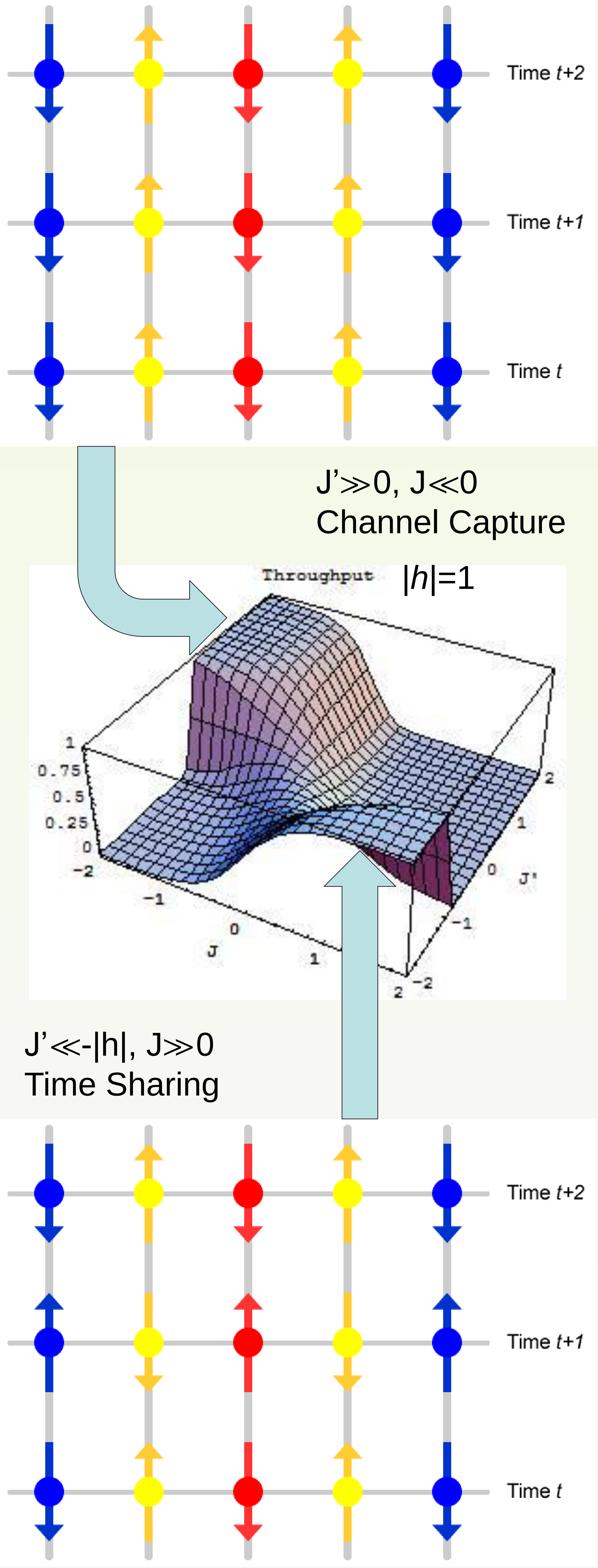
Collision Channel:
J'≫0, h≪0, J≪0, h>≈2J
Multipacket Reception Channel:
- J'≫0, J≪0
- J'≪-|h|, J≫0

Collision Channel



J'≫0, h≪0, J≪0, h>≈2J
Only favors transmissions when both neighbors are idle.

Multipacket Reception Channel



J'≫0, J≪0
Channel Capture

J'≪-|h|, J≫0
Time Sharing