Practical Codes Over Packet Networks

Overlapped Chunked Codes: Computationally Efficient Network Codes

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1 Goal

To design a new coding scheme which is *capacity-achieving* and *computationally efficient* over line networks (with erasure channels) with *arbitrary traffic*.

Capacity-Achieving: *The transmission rate approaches the network capacity asymptotically.*
Computationally Efficient: *The encoding, recoding and decoding algorithms require the least possible number of operations.*

2 Network Model

Line Networks: Basic of any multicast scenario in a worst-case analysis.

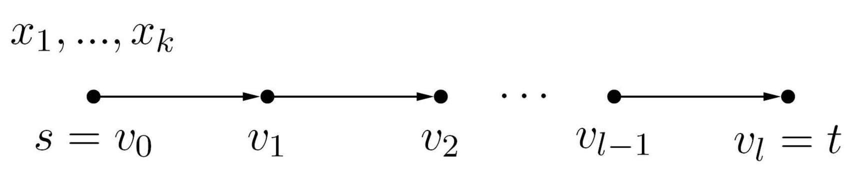

Figure 1: A line network of length l, the source s and the terminal t.

Traffic (The Set of Successful Transmissions): Any arbitrary schedule of a given capacity (The accurate modeling is often too complex and/or infeasible) [3].

1. Unconstrained delays and/or reordering of packets [4].
2. Feedback solution: additional delay or too difficult to implement [2].

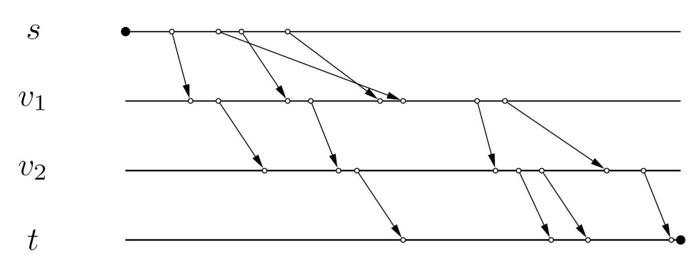

Figure 2: An instance of a schedule of capacity $n = 4$ over a line network of length $l = 3$.

3 Problem

To give an upper bound (for a worst case scenario) on the capacity of any arbitrary schedule (n) on which a given code transmits $k \leq n$ information symbols with probability of failure ϵ.

4 Existing Solutions

4.1 Dense Codes (Random Network Codes) [1]

Encoding: Random linear combination of information symbols ($O(k)$).
Recoding: Random linear combination of received packets ($O(k)$).
Decoding: Solve a set of dense linear equations ($O(k)$).

4.2 Chunked Codes (CC)[3]

Main Idea: Applying a Dense Code on *chunks* (smaller sub-messages of the original message) requires **less operations** [4].

Chunking: Partition k information symbols into q disjoint chunks of k/q contiguous symbols.

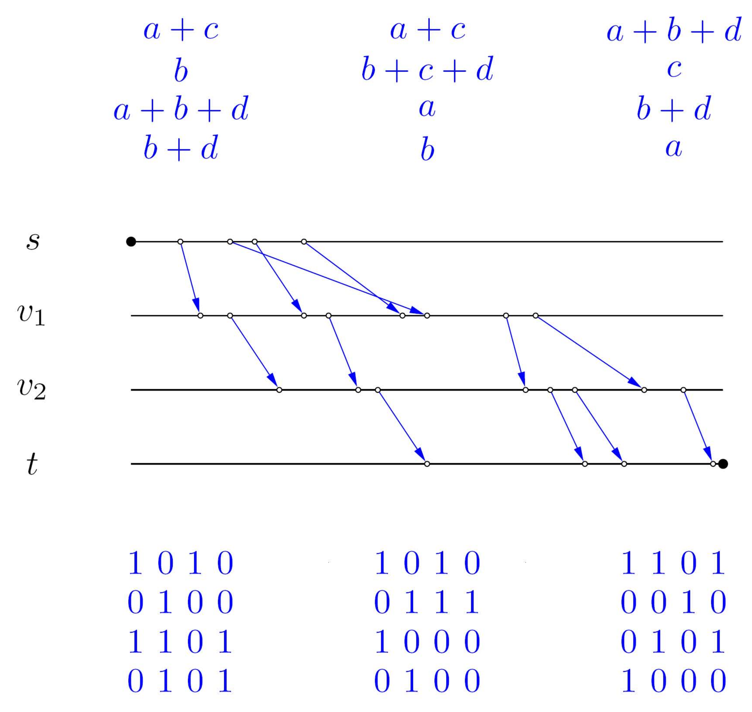

Figure 3: An instance of dense coding on the schedule depicted in Fig. 2, where $k = 4$, i.e., the information symbols are $a, b, c,$ and d.

Encoding/Recoding: Each node randomly chooses a chunk at each time instant and sends a packet associated with this chunk by a Dense Code ($O(k/q)$).

Decoding: Solve each dense sub-matrix associated with a chunk separately ($O(k/q)$).

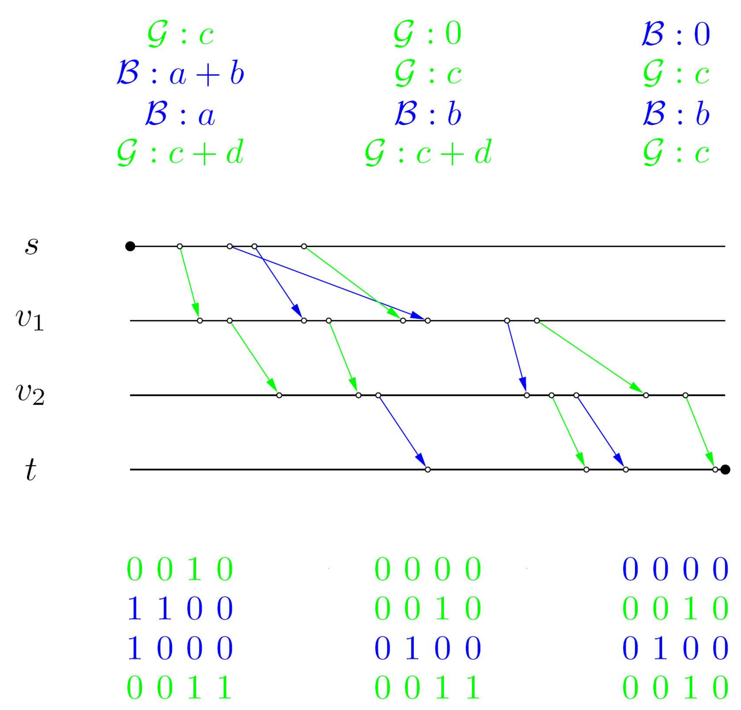

Figure 4: An instance of chunked coding with $q = 2$ (two chunks $\mathcal{B} = \{a, b\}$ and $\mathcal{G} = \{c, d\}$) on the schedule depicted in Fig. 2, where $k = 4$.

4.3 Motivation

Dense Codes: Not efficient from a computational complexity perspective
Chunked Codes: Each chunk has to be decoded separately, i.e., its associated sub-matrix has to be full-rank → the need for a **larger overhead**!

Question: Are there any matrices with a block diagonal structure having better rank property?
Answer: Yes, let the chunks do overlap.

4.4 Intuition [5]

Conjecture 4.1 *Suppose* ***any two contiguous chunks overlap*** *in γ symbols. For any $\gamma > \sqrt{k}$, the block diagonal matrix at the terminal has the* ***rank property similar to a fully random matrix****, regardless of the rank of each sub-matrix associated with a chunk.*

5 Proposed Solution: Overlapped Chunked Codes (OCC)

Chunking: Partition k symbols into q chunks, each of size α, where any two contiguous chunks overlap by $\gamma = \alpha - k/q$ symbols.
Encoding/Recoding: Each node randomly chooses a chunk to operate on by a Dense Code ($O(\alpha)$).
Decoding: Solve a block diagonal matrix ($O(\alpha)$).

6 Analysis: Worst-Case Performance

6.1 Analytical Results

k information symbols can be delivered to the terminal with probability of failure no larger than ϵ over a line network of length l and under a schedule of capacity n, so long as

Theorem 6.1 *Dense Code:*

$$n > k + l \log kl/\epsilon + \log 1/\epsilon + l + 1$$

Moreover, the encoding and decoding costs are each $O(k)$.

Theorem 6.2 *Chunked Code:*

$$n > k + ql \log kl/\epsilon + q \log 1/\epsilon + q \log q + q$$

provided that

$$l^4 q^2 \log \frac{kl}{\epsilon} = o(k) \,. \qquad (1)$$

Furthermore, the encoding and decoding costs are each $O(k/q)$.

Theorem 6.3 *Overlapped Chunked Code:*

$$n > k + ql \log kl/\epsilon + ql + \log 1/\epsilon + 1,$$

provided that $\gamma \geq \sqrt{k}$ and the condition (1) is met.

6.2 Comparison

1. Asymptotically, CC and OCC have a similar speed of convergence, i.e., $O(ql \log kl/\epsilon)$, **but for finite-length codes OCC provides a larger speed.**
2. **For a fixed q, OCC requires less overhead compared to CC**, but at the cost of slightly increasing the computational complexity.
3. **OCC provides a better tradeoff between the speed of convergence and the coding costs.**
4. For a given coding cost, OCC asymptotically achieves the capacity with a larger overhead.

7 Simulation Results: Average Performance

7.1 Objectives

1. To investigate the probability of successful decoding as a function of the network capacity
2. To investigate how this function changes with l, q, and the overlap γ.

7.2 Setup

1. Networks of length $l \in \{2, 4\}$ are simulated with randomly generated schedules of capacity n, for integers $n \in \{1024, ..., 1984\}$.
2. $k = 1024$, where each symbol is assumed to be a binary random variable.
3. For different values of n and l, the coding schemes are applied on random schedules until we have 100 successful decoding events.
- q is set to 2 or 4 in CC and 4 in OCC.

7.3 Comparison

1. **For a fixed q, OCC is always superior to CC.**
2. **For a given coding cost, OCC still outperforms CC.**

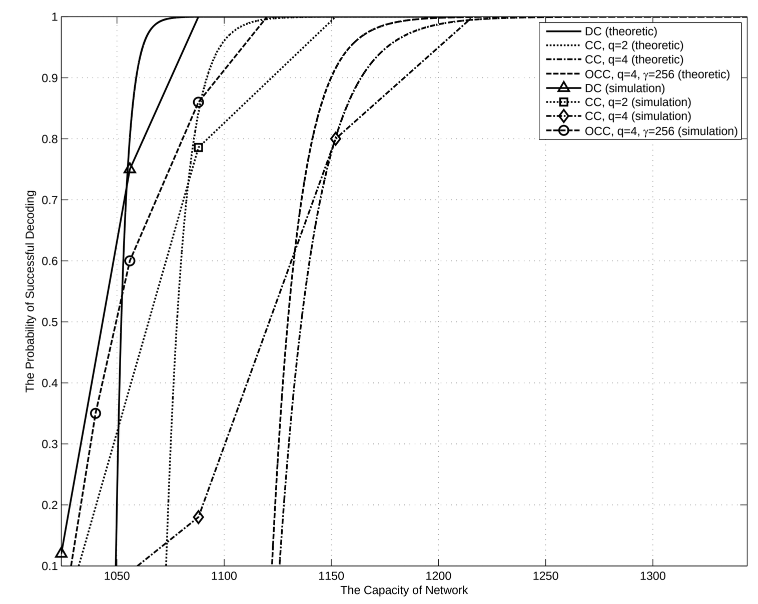

Figure 5: The performance comparison over line networks with $l = 2$.

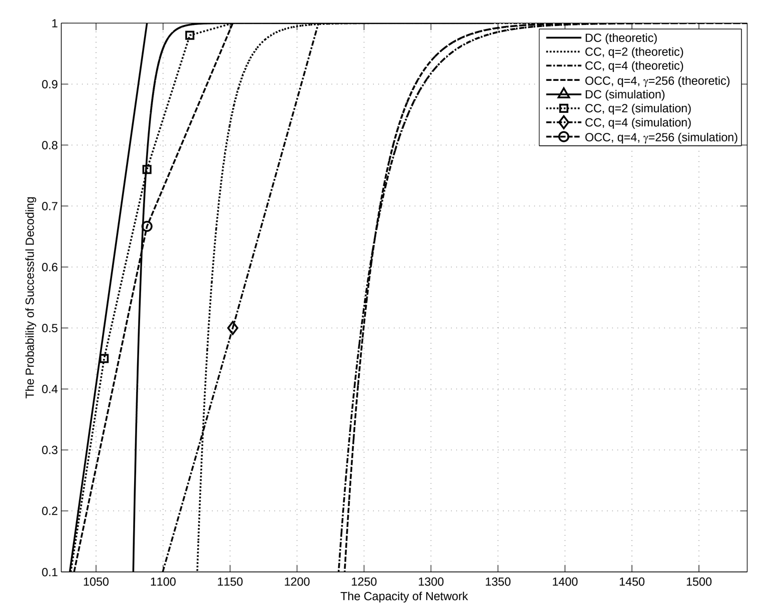

Figure 6: The performance comparison over line networks with $l = 4$.

References

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