

The Inclusion-Exclusion Formula

$$\begin{aligned}\mu\left(\bigcup_{k=1}^m A_k\right) &= \sum_{1 \leq i \leq m} \mu(A_i) - \sum_{1 \leq i < j \leq m} \mu(A_i \cap A_j) + \cdots \\ &\quad + (-1)^{m+1} \mu(A_1 \cap A_2 \cap \cdots \cap A_m).\end{aligned}$$

Theorem 3.19 (Variation of the Inclusion-Exclusion Formula)

$$\begin{aligned}\mu\left(\bigcap_{k=1}^m A_k - B\right) &= \sum_{1 \leq i \leq m} \mu(A_i - B) - \sum_{1 \leq i < j \leq m} \mu(A_i \cup A_j - B) + \cdots \\ &\quad + (-1)^{m+1} \mu(A_1 \cup A_2 \cup \cdots \cup A_m - B).\end{aligned}$$

Proof See textbook.